1. Examine the following optimization problem. State the total number of variables and list them. State the number of independent variables, and list a set.

$$\begin{aligned} \text{Minimize: } & f(x) = 4x_1 - x_2^2 - 12 \\ \text{subject to: } & 25 - x_1^2 - x_2^2 = 0 \\ & 10x_1 - x_1^2 + 10x_2 - x_2^2 - 34 \geq 0 \\ & (x_1 - 3)^2 + (x_2 - 1)^2 \geq 0 \\ & x_1, x_2 \geq 0 \end{aligned}$$

2. A certain gas contains moisture, which you need to remove by compression and cooling so that the gas will finally contain not more than 1% moisture (by volume). The respective cost of compression and cooling equipments are $p^{1.4}$ and $(350 - T)^{1.9}$, where p and T are pressure (psi) and temperature (K), respectively. What is the best temperature to use? Define the independent and the dependent variables, the constraints and the objective function. Formulate the complete optimization problem and you do not have to solve the problem but list all the steps to solve it.

3. The chemical plant is producing three products (E, F, G) from the limited supply of three raw materials (A, B, C). Each product is produced in separate process (1, 2, 3) as follow

Process 1: $A + B \rightarrow E$ Process 2: $A + B \rightarrow F$ Process 3: $3A + 2B + C \rightarrow G$

Formulate the objective function to maximize the total operating profit per day by using the following process data.

Raw Materials	Maximum availability, kg/day	Cost per kg
A	$40,000$	1.5
B	$30,000$	2.0
C	$25,000$	2.5

Process	Product	Reactant requirements,kg/kg	Processing cost per kg	Selling price per kg
1	E	$\frac{2}{3}A, \frac{1}{3}B$	1.5	4.0
2	F	$\frac{2}{3}A, \frac{1}{3}B$	0.5	3.3
3	G	$\frac{1}{2}A, \frac{1}{6}B, \frac{1}{3}C$	1.0	3.8

4. Are the following function continuous (specify the range) or discrete? (a) $f(x) = 1/x$, (b) $f(x) = \ln x$, (c) $f(x) = e^x$, (d) $f(x) = ax_{n-1} + b(x_0 - x_n)$ and (e) $f(x) = (x_D - x_s)/(1 + x_s)$. In each case, specify the range of x for which $f(x)$ and $f'(x)$ are continuous.

5. Determine whether the following function $f(x)$ is convex, concave, strictly convex, strictly concave, all or none these classes in the range $-\infty \leq x \leq \infty$. (a) $f(x) = 3x^2$, (b) $f(x) = 2x$, (c) $f(x) = -5x^2$, and (d) $f(x) = 2x^2 - x^3$.