Assignment/Tutorial Sheet – 03

1. Sketch the objective function and constraints of the following optimization problems and identify the feasible and infeasible regions. Is $x = [1\ 1]^T$ an interior, boundary or exterior point in these problems.

$$\begin{aligned}
&\text{(a) Minimize:} && f(x) = 2x_1^2 - 2x_1x_2 + 2x_2^2 - 6x_1 + 6 \\
&\text{subject to} && g_1(x) = x_1 + x_2 \le 2 \\
&\text{(b) Minimize:} && f(x) = 2x_1^3 - 3x_1x_2 + 4 \\
&\text{subject to} && g_1(x) = 5x_1 + 2x_2 \ge 18 \\
& && h_1(x) = -2x_1 + x_2^2 = 5
\end{aligned}$$

2. Given the following single variable functions (a) $f(x) = x^5 + x^4 - (x^3/2) + 2$ and (b) $f(x) = (2x+1)^2(x-4)$. Determine, for each of the above functions, the following: (i) Regions where the function is increasing and decreasing (ii) Inflection points, if any (iii) Regions where the function is concave and convex (iv) Local and global maxima, if any (v) Local and global minima, if any.

3. Determine the gradient and Hessian matrix for the objective function $f(x) = 10(x_2 - x_1^2)^2 + (1 - x_1)^2$ at the point $x_k = (0, 1)$.

4. Determine the convexity of the following functions? Are they strictly convex/concave? Why? (a) $2x_1^2 + 2x_1x_2 + 3x_2^2 + 7x_1 + 8x_2 + 25$, (b) e^{5x}, (c) $e^{x_1} + e^{x_2}$, (d) $|x|$ and (e) $x_1^2 + x_2^2 + x_3^2$.

5. Classify the following matrices as (a) positive-definite, (b) negative-definite (c) neither

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 0.1 & 0 \\ 3 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix} \quad E = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

6. In designing a can to hold a specified amount of drink, the cost function (to be minimized) for manufacturing one can is $f(D, h) = \pi(Dh) + \pi(D^2/2)$ and the constraints are $(\pi/4)D^2h \ge 400$, $3.5 \le D \le 8$ and $8 \le h \le 18$. Based on the optimization problem, answer the following with the use of mathematics support to your statement. (a) State whether $f(D, h)$ is unimodel (one extremum) or multimodel (more then one extremum) (b) State whether $f(D, h)$ is continuous or not, (c) State whether $f(D, h)$ is convex, concave or neither, (d) State whether $f(D, h)$ alone meets the necessary and sufficient conditions for a minimum to exist, and (e) State whether the constraints form a convex region.